POLITECNICO DI TORINO
Repository ISTITUZIONALE

Testing Bell's inequality using ballistic electrons in semiconductors

Original
Testing Bell's inequality using ballistic electrons in semiconductors / IONICIOIU R.; ZANARDI P.; ROSSI F.. - In: PHYSICAL REVIEW A. - ISSN 1050-2947. - 63:5(2001), pp. 50101-1-50101-4.

Availability:
This version is available at: 11583/1405231 since:

Publisher:
APS American Physical Society

Published
DOI:10.1103/PhysRevA.63.050101

(Article begins on next page)

05 August 2020

PHYSICAL REVIEW A, VOLUME 63, 050101(R)

Testing Bell's inequality with ballistic electrons in semiconductors

Radu Ionicioiu,[1,2] Paolo Zanardi,[1,2] and Fausto Rossi[1,2,3]
[1]*Istituto Nazionale per la Fisica della Materia (INFM), UdR Torino-Politecnico, 10129 Torino, Italy*
[2]*Institute for Scientific Interchange (ISI), villa Gualino, Viale Settimio Severo 65, I-10133 Torino, Italy*
[3]*Dipartimento di Fisica, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*
(Received 21 December 2000; published 10 April 2001)

We propose an experiment to test Bell's inequality violation in condensed-matter physics. We show how to generate, manipulate, and detect entangled states using ballistic electrons in Coulomb-coupled semiconductor quantum wires. Due to its simplicity (only five gates are required to prepare entangled states and to test Bell's inequality), the proposed semiconductor-based scheme can be implemented with currently available technology. Moreover, its basic ingredients may play a role towards large-scale quantum-information processing in solid-state devices.

DOI: 10.1103/PhysRevA.63.050101 PACS number(s): 03.65.Ta, 03.67.Lx, 85.35.Be, 73.23.Ad

The introduction of quantum information processing (QIP) [1] has led, on the one hand, to unquestionable intellectual progress in understanding basic concepts of information or computation theory; on the other hand, this has stimulated new thinking about how to realize QIP devices able to exploit the additional power provided by quantum mechanics. Such novel communication and computation capabilities are primarily related to the ability of processing *entangled states* [1]. To this end, one should be able to perform precise quantum-state synthesis, coherent quantum manipulations (*gating*) and detection (*measurement*). The unavoidable interaction of any realistic quantum system with its environment tends to destroy coherence between quantum superpositions. Thus, decoherence modifies the above ideal scenario and imposes further strong constraints on candidate systems for QIP. Indeed, mainly due to the need of low decoherence rates, the only experimental realizations of QIP devices originated in atomic physics [2] and in quantum optics [3]. It is, however, generally believed that any large-scale application of QIP cannot be easily realized with such quantum hardware, which does not allow the scalability of existing microelectronics technology. In contrast, in spite of the relatively strong decoherence, a solid-state implementation of QIP can benefit synergistically from the recent progress in single-electron physics [4], as well as in nanostructure fabrication and characterization [5].

As already mentioned, the key ingredient for computational speedup in QIP is entanglement. Einstein-Podolsky-Rosen (EPR) pairs [6] and three-particle Greenberger-Horne-Zeilinger (GHZ) states [7] are at the heart of quantum cryptography, teleportation, dense coding, entanglement swapping, and of many quantum algorithms. Experimentally, two-particle entangled states have been prepared using photons [8] and trapped ions [9]; only recently has a photonic three-particle entangled state (GHZ) been also measured [10]. A few proposals for the generation of entangled states in solid-state physics have been recently put forward [11–18], but to date there are no experimental implementations.

In this Rapid Communication we propose an experiment to test Bell's inequality violation in condensed-matter physics. More specifically, we shall show how to generate, manipulate, and measure entangled states using ballistic electrons in coupled semiconductor quantum waveguides (quantum wires). As we shall see, our scheme allows for a direct test of Bell's inequality in a solid-state system. To this end, a relatively simple gating sequence (five gates only) is identified.

The proposed experimental setup is based on the semiconductor quantum hardware of the earlier proposal for *quantum computation with ballistic electrons* by Ionicioiu *et al.* [19]. We summarize in the following the main features of this proposal, which has been recently analyzed and validated through numerical simulations by Bertoni *et al.* [20].

The main idea is to use ballistic electrons as *flying qubits* in semiconductor quantum wires (QWRs). In view of the nanometric carrier confinement reached by current fabrication technology [5], state-of-the-art QWRs behave as quasi-one-dimensional electron waveguides. Due to the relatively large intersubband energy splittings as well as to the good quality of semiconductor/semiconductor interfaces, electrons within the lowest QWR subband at low temperature may experience extremely high mobility. In such conditions their coherence length can reach values of a few microns; therefore, on the nanometric scale electrons are in the so-called *ballistic regime* and the phase coherence of their wave functions is preserved. This coherent-transport regime is fully compatible with existing semiconductor nanotechnology [5] and has been the natural arena for a number of interferometric experiments with ballistic electrons [21,22]. Such a fully coherent regime is the basic prerequisite for any QIP.

The building block of our quantum hardware is a pair of adjacent QWR structures. The qubit state is defined according to the quantum-mechanical state of the electron across this two-wire system. More precisely, we shall use the so-called *dual-rail* representation for the qubit: we define the basis state $|0\rangle$ by the presence of the electron in one of the wires (called the 0 rail) and the basis state $|1\rangle$ by the presence of the electron in the other one (the 1 rail). Saying that the electron is in a given wire we mean that (i) its wave function is localized on that QWR and (ii) its free motion along the wire is well described in terms of a quasimonoenergetic wave packet within the lowest QWR electron subband (with central kinetic energy E and central wave vector $k = \sqrt{2m^*E}/\hbar$).

An appealing feature of the proposed scheme is the mobile character of our qubits: using flying qubits we can transfer entanglement from one place to another, without the need to interconvert stationary into mobile qubits. In the case of stationary qubits (e.g., electron spins in quantum dots) this is not easily done.

Any QIP device can be built using only single- and two-qubit gates [23]. We choose the following set of universal quantum gates: $\{H, P_\varphi, P_\pi^C\}$, where

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

is a Hadamard gate, $P_\varphi = \mathrm{diag}(1, e^{i\varphi})$ is a single-qubit phase shift, and P_π^C is a controlled sign flip. We shall use the more general two-qubit gate $P_\varphi^C = \mathrm{diag}(1,1,1,e^{i\varphi})$.

We now briefly describe the physical implementation of the universal quantum gates in terms of the previously introduced dual-rail representation. The Hadamard gate can be implemented using an *electronic beam splitter*; also called *waveguide coupler* [24–26]. The idea is to design the two-wire system in such a way as to spatially control the interwire electron tunneling. For a given interwire distance, a proper modulation (along the QWR direction) of the interwire potential barrier can produce a linear superposition of the basis states $|0\rangle$ and $|1\rangle$. More specifically, let us consider a *coupling window*, i.e., a tunneling-active region, of length L_c characterized by an interwire tunneling rate $\omega = 2\pi/\tau$. As it propagates, the electron wave packet oscillates back and forth between the two waveguides with frequency ω. Let $v = \hbar k/m^*$ be the group velocity of the electron wave packet along the wire; then, the state $|0\rangle$ goes into the superposition $\cos\alpha|0\rangle + \sin\alpha|1\rangle$ with $\alpha = \omega t = 2\pi L/v\tau$. Let L_t be the length necessary for the complete transfer of the electron from one wire to the other, $\alpha = \pi$, $L_t = v\tau/2$. For a coupling length $L_c = L_t/2$ the device is equivalent to a beam splitter, and hence, up to a phase shift, to a Hadamard gate. By a proper modulation of the interwire potential barrier we can vary the tunneling rate ω and therefore the rotation angle α. As a result, this structure can operate as a NOT gate by adjusting the interwire potential barrier such that $L_c = L_t$ (π rotation). Similarly, the gate can be turned off by an appropriate potential barrier for which the electron wave packet undergoes a full oscillation period, returning back to its original state ($L_c = 2L_t$, 2π rotation). Another way of turning the H gate off is to suppress interwire tunneling by applying a strong potential bias to the coupled QWR structure.

The phase shifter P_φ can be implemented using either a potential step (with height smaller than the electron energy $V < E$) or a potential well along the wire direction; the well is preferred since the phase shift induced is more stable under voltage fluctuations. In order to have no reflection from the potential barrier, the width L of the barrier should be an integer multiple of the half wavelength of the electron in the step and/or well region, $L = n\lambda/2$, $n \in \mathbb{N}$.

We finally describe the two-qubit gate. In our scheme the controlled phase shifter P_φ^C is implemented using a *Coulomb coupler* [27]. This quantum gate exploits the Coulomb interaction between two single electrons in different QWR pairs (representing the two qubits). The gate is similar in construction to the beam splitter previously introduced. In this case, the multiwire structure (see Fig. 4) needs to be tailored in such a way as (i) to obtain a significant Coulomb coupling between the two 1 rails only and (ii) to prevent any single-particle interwire tunneling. Therefore, only if both qubits are in the $|1\rangle$ state do they both experience a phase shift induced by the two-body Coulomb interaction. In contrast, if at least one qubit is in the $|0\rangle$ state, then nothing happens.

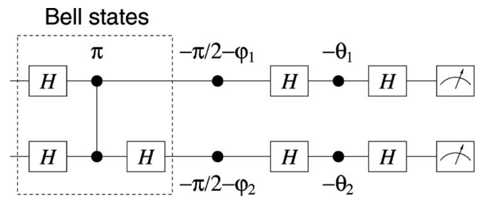

FIG. 1. Quantum network for the measurement of Bell's inequality. Bell states are prepared in the dashed box; then the first qubit is measured along the direction $\mathbf{a} = (\theta_1, \varphi_1)$, and the second qubit along the direction $\mathbf{b} = (\theta_2, \varphi_2)$.

The proposed quantum hardware has some advantages. First, the QIP device needs not to be "programmed" at the hardware level (by burning off the gates), as it may appear. Programming is done by switching on or off the gates and this way any quantum algorithm can be implemented [28]. Second, we use *cold programming*, i.e., we set all the gates before "launching" the electrons, so we do not need ultrafast (i.e., subdecoherent) electronics for gate operations. This property is essential and is a distinct advantage of the proposed quantum architecture over other solid-state proposals [29]. Therefore, the gating sequence needed for the proposed experiment can be preprogrammed using *static electric fields only*

One important requirement of our quantum hardware is that electrons within different wires need to be synchronized at all times in order to properly perform two-qubit gating (the two electron wave packets should simultaneously reach the Coulomb-coupling window). It is thus essential to have highly monoenergetic electrons launched simultaneously. This can be accomplished by properly tailored energy filters and synchronized single-electron injectors at the preparation stage.

We now turn to the proposed experimental setup for testing Bell's inequality. Two-particle entangled states (Bell states) can be generated using three Hadamard gates and a controlled-sign shift (see dashed box in Fig. 1; the controlled sign shift plus the lower two Hadamard gates form a controlled-NOT gate). Consider the correlation function for two (pseudo)spins $P(\mathbf{a},\mathbf{b}) = \langle \sigma_{\mathbf{a}}^{(1)} \sigma_{\mathbf{b}}^{(2)} \rangle$ (here, $\sigma_{\mathbf{a}} = \sigma_i a_i$ is the pseudospin projection along the unit vector $\mathbf{a}$) [30]. Any local, realistic hidden-variable theory obeys the Bell-CHSH (Clauser-Horne-Shimony-Holt) [31,32] inequality:

$$|P(\mathbf{a},\mathbf{b}) + P(\mathbf{a}',\mathbf{b}) + P(\mathbf{a}',\mathbf{b}') - P(\mathbf{a},\mathbf{b}')| \leq 2. \tag{1}$$

This inequality is violated in quantum mechanics. For the singlet $|\Psi^-\rangle$, a standard calculation gives the result

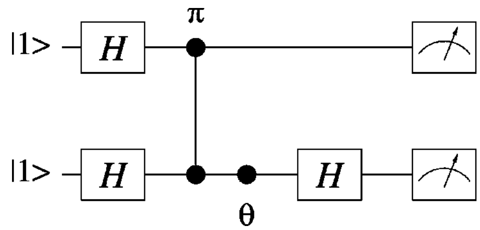

FIG. 2. Testing Bell's inequality for the singlet state $|\Psi^-\rangle$. The quantum network is obtained from Fig. 1 by setting $\varphi_1=\varphi_2=-\pi/2$, $\theta_1=0$ and relabeling $\theta=-\theta_2$.

$$P(\mathbf{a},\mathbf{b})\equiv\langle\Psi^-|\sigma_{\mathbf{a}}^{(1)}\sigma_{\mathbf{b}}^{(2)}|\Psi^-\rangle=-\mathbf{a}\cdot\mathbf{b}. \tag{2}$$

Choosing $\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}'=\mathbf{a}'\cdot\mathbf{b}'=-\mathbf{b}'\cdot\mathbf{a}=\sqrt{2}/2$, we obtain $2\sqrt{2}\leqslant 2$, violating thus Bell inequality (1).

Let us now focus on the correlation function $P(\mathbf{a},\mathbf{b})$. In the EPR-Bohm gedankenexperiment we need to measure the spin component of one particle along a direction $\mathbf{n}$. However, in our setup this is not directly possible, since we can measure only σ_z, i.e., whether the electron is in the 0 or in the 1 rail. The solution is to do a unitary transformation $|\psi\rangle\rightarrow|\psi'\rangle=U|\psi\rangle$, such that the operator $\sigma_{\mathbf{n}}$ is diagonalized to σ_z, $\langle\psi|\sigma_{\mathbf{n}}|\psi\rangle=\langle\psi'|\sigma_z|\psi'\rangle$. We are looking for a unitary transformation U that satisfies $U^+\sigma_z U=\sigma_{\mathbf{n}}$, with $\mathbf{n}=(\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta)$ a unit vector. In terms of our elementary gates we obtain $U(\theta,\varphi)=HP_{-\theta}HP_{-\varphi-\pi/2}$.

Thus, measuring the spin (in the EPR-Bohm setup) along a direction $\mathbf{n}$ is equivalent to performing the unitary transformation $U(\theta,\varphi)$ followed by a measurement of σ_z. Going back to our entangled pair, we now apply on each qubit a local transformation $U(\theta_1,\varphi_1)$ and $U(\theta_2,\varphi_2)$, respectively. Here, $\mathbf{a}=(\theta_1,\varphi_1)$ and $\mathbf{b}=(\theta_2,\varphi_2)$ are the two directions discussed above, at the very end, we measure σ_z (i.e., electron in 0 or in 1 rail; see Fig. 1).

For the singlet $|\Psi^-\rangle$ the correlation function depends only on the scalar product of the two directions (2), and hence only on the angle between them. Without loss of generality, we can choose $\varphi_1=\varphi_2=-\pi/2$, $\theta_1=0$ and relabel $\theta=-\theta_2$. Since $H^2=\mathbb{1}$, the gating sequence simplifies to only five gates, as shown in Fig. 2. With this simple network we can measure the correlation function (2) that violates Bell's inequality (1). To perform an Aspect-type experiment [8], we have to independently choose the directions of measurement for each qubit after the electrons are entangled. In this case we need three more gates (after the P_π^C gate) $HP_{-\theta_1}H$ on the upper qubit in Fig. 2.

In practice the situation is more complex. The essential ingredient for producing entanglement is the controlled-sign shift gate P_π^C that involves an interaction between the two qubits. Experimentally this requires a good timing of the two electrons (they should simultaneously reach the two-qubit gating region). Suppose that instead of having an ideal P_π^C gate preparing an ideal singlet (dashed box in Fig. 1), in practice we realize a P_α^C gate (possible with unknown phase α). In this case, instead of preparing the singlet $|\Psi^-\rangle$, we end up with the following state:

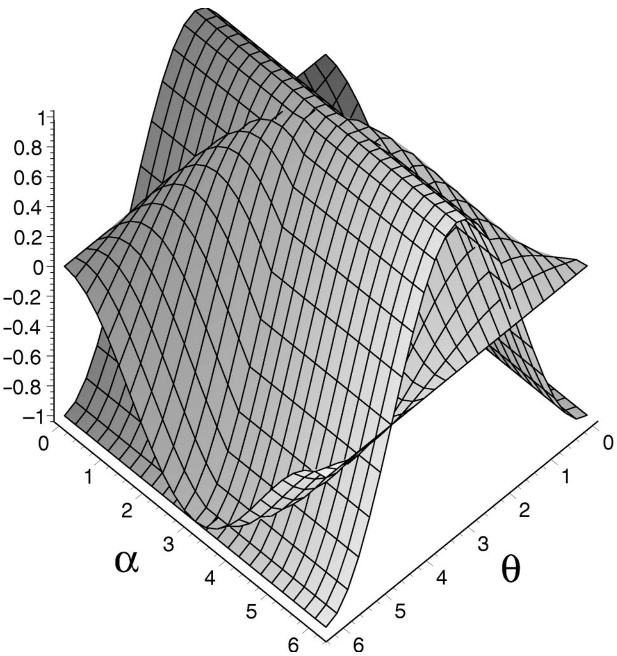

FIG. 3. Correlation functions $S(\theta)$ and $S(\alpha,\theta)$ for the ''ideal'' and ''realistic'' singlet, respectively; note that $S(\pi,\theta)=S(\theta)$.

$$|\Psi_\alpha\rangle=|\Psi^-\rangle+e^{i\alpha/2}\cos\frac{\alpha}{2}\,\frac{|1\rangle(|0\rangle-|1\rangle)}{\sqrt{2}}, \tag{3}$$

which is a superposition of the singlet and of a separable state.

Let us now consider the experimental setup discussed above, with $\mathbf{a}=(0,\sin\theta_1,\cos\theta_1)$ and $\mathbf{b}=(0,\sin\theta_2,\cos\theta_2)$, both in the Oyz plane. For the correlation function of the imperfect singlet $|\Psi_\alpha\rangle$ we obtain

$$S(\alpha,\theta)\equiv\langle\Psi_\alpha|\sigma_{\mathbf{a}}^{(1)}\sigma_{\mathbf{b}}^{(2)}|\Psi_\alpha\rangle=-\sin\frac{\alpha}{2}\sin\left(\theta+\frac{\alpha}{2}\right), \tag{4}$$

with $\theta=\theta_2-\theta_1$. For $\alpha=\pi$ we recover the correlation function of the singlet, $S(\theta)\equiv S(\pi,\theta)=-\cos\theta$. The two functions are plotted in Fig. 3; $S(\theta)$ can be identified by noting that there is no α dependence.

Experimentally, since the one-qubit gate P_θ is easier to control, we can measure the coupling α of the Coulomb coupler P_α^C by measuring the dependence of the correlation function $S(\alpha,\theta)$ on the phase shift θ (which can be accurately determined). This procedure can be used to determine the purity of the singlet, and hence to test and calibrate the Coulomb coupler.

We are now interested to see how small the coupling α can be in order to still have a violation of Bell's inequality. The question we ask is: *For what values of α does the correlation function $S(\alpha,\theta)$ in Eq. (4) violate Bell's inequality in Eq. (1)?* To this end, we have found a numerical solution: the inequality (1) is violated for $\alpha\in(\pi/2,3\pi/2)$.

A schematic representation of the proposed experimental setup for measuring Bell's inequality violation is presented in Fig. 4. It is possible to reduce the number

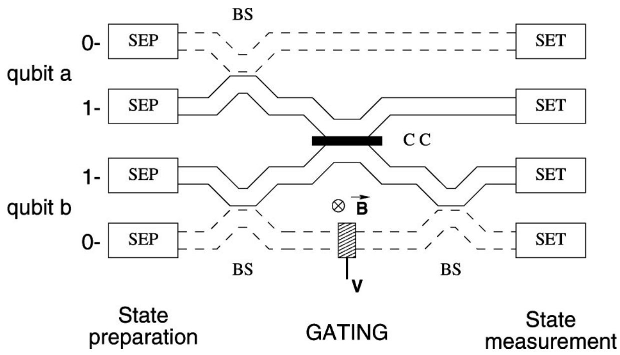

FIG. 4. Experimental setup to test the Bell-CHSH inequality; the 0 rails of each qubit are dashed for clarity. A potential V applied on top of the 0 rail (dashed box in the figure) is used to produce a phase shift $P^0_{-\theta}$ on the second qubit; alternatively, the same effect can be achieved with a magnetic field $\vec{B}$ (via the Aharonov-Bohm effect).

of gates on the 1 rail by using a phase shifter on the 0 rail $P^0_{-\theta} \equiv \mathrm{diag}(e^{-i\theta}, 1)$ instead of the 1 rail one used so far $P^1_\theta \equiv \mathrm{diag}(1, e^{i\theta})$, since the two are equivalent (up to an overall phase) $P^1_\theta = e^{i\theta} P^0_{-\theta}$.

In our setup there are two different ways of producing a phase shift P_θ: (i) electrically, with a potential applied on top of the 0 rail (the quantum well described above); (ii) magnetically, via the Aharonov-Bohm effect, by applying locally a magnetic field on the area between the lower two beam splitters (this can be done since the P_θ and P^C_α gates commute). The second method has the advantage of avoiding the no-reflection condition for the potential well (the length of the gate should be a half integer multiple of the electron wavelength). Either way can be used experimentally.

We stress that Aharonov-Bohm rings and quantum interference experiments with ballistic electrons are standard tools in mesoscopic physics. A two-slit experiment with an Aharonov-Bohm ring having a quantum dot embedded in one arm has been reported in [21,22]. This experiment is similar to the layout of the lower qubit in Fig. 4, but the authors do not use beam splitters and Coulomb couplers. In the experimental setup presented here, the more difficult part will be to implement the Coulomb coupler and to perform the experiment at the single electron level. In our case, preparation and measurement of the states are done using single electron pumps and single electron transistors [33], respectively.

In conclusion, we have proposed an alternative measurement of Bell's inequality violation in coupled semiconductor nanostructures using ballistic electrons. Due to the relative simplicity of the proposed experimental setup (only five gates are needed to produce entanglement and to test Bell's inequality), this measurement scheme is potentially feasible in terms of current semiconductor nanotechnology.

R.I. is grateful to G. Amaratunga, F. Udrea, and A. Popescu for stimulating discussions and financial support.

[1] D. P. DiVincenzo and C. Bennett, Nature (London) **404**, 247 (2000); A. Steane, Rep. Prog. Phys. **61**, 117 (1998).
[2] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995); A. Sorensen and K. Molmer, *ibid.* **82**, 1971 (1999).
[3] Q. A. Turchette *et al.*, Phys. Rev. Lett. **75**, 4710 (1995); A. Imamoglu *et al.*, *ibid.* **83**, 4204 (1999).
[4] K. Likharev, Proc. IEEE **87**, 606 (1999).
[5] L. Pfeiffer *et al.*, Microelectron. J. **28**, 817 (1997).
[6] A. Einstein *et al.*, Phys. Rev. **47**, 777 (1935).
[7] D. M. Greenberger *et al.*, Am. J. Phys. **58**, 1131 (1990).
[8] A. Aspect *et al.*, Phys. Rev. Lett. **49**, 1804 (1982).
[9] C. Monroe *et al.*, Science **272**, 1131 (1996); D. J. Wineland *et al.*, Proc. R. Soc. London **A454**, 411 (1998).
[10] D. Bouwmeester *et al.*, Phys. Rev. Lett. **82**, 1345 (1999).
[11] D. DiVincenzo and D. Loss, e-print cond-mat/9901137.
[12] G. Burkard *et al.*, Phys. Rev. B **61**, R16303 (2000); e-print cond-mat/9906071.
[13] D. Loss and E. V. Sukhorukov, Phys. Rev. Lett. **84**, 1035 (2000); e-print cond-mat/9907129.
[14] D. DiVincenzo *et al.*, e-print cond-mat/9911245.
[15] D. DiVincenzo, e-print quant-ph/0002077.
[16] L. Quiroga and N. F. Johnson, Phys. Rev. Lett. **83**, 2270 (1999); e-print cond-mat/9901201.
[17] J. H. Reina *et al.*, Phys. Rev. A **62**, 012305 (2000); e-print quant-ph/9911123.
[18] C. Barnes *et al.*, Phys. Rev. B **62**, 8410 (2000); e-print cond-mat/0006037.
[19] R. Ionicioiu *et al.*, Int. J. Mod. Phys. B **15**, 125 (2001); e-print quant-ph/9907043.
[20] A. Bertoni *et al.*, Phys. Rev. Lett. **84**, 5912 (2000).
[21] A. Yacoby *et al.*, Phys. Rev. Lett. **74**, 4047 (1995).
[22] R. Schuster *et al.*, Nature (London) **385**, 417 (1997).
[23] A. Barenco *et al.*, Phys. Rev. A **52**, 3457 (1995).
[24] J. A. del Alamo and C. C. Eugster, Appl. Phys. Lett. **56**, 78 (1990).
[25] N. Tsukada *et al.*, Appl. Phys. Lett. **56**, 2527 (1990).
[26] M. J. Liang, G. G. Siu, and K. S. Chan, J. Phys. D **27**, 1513 (1994).
[27] M. Kitagawa and M. Ueda, Phys. Rev. Lett. **67**, 1852 (1991).
[28] The phase shifter P_φ can be switch on/off by simply switching on or off the electric bias applied to the gate. Since $P^C_{2\pi} = \mathbb{1}$, this gate can also be turned on or off by means of external biases and appropriate design.
[29] Any quantum hardware using ''stationary qubits'' needs ultrafast electronics or optical pulses to perform (subdecoherent) gate operations on picosecond time scales.
[30] We stress that we are dealing with charge degrees of freedom and the present pseudospin notation has nothing to do with the physical spin of the electrons.
[31] J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1964).
[32] J. F. Clauser *et al.*, Phys. Rev. Lett. **23**, 880 (1969).
[33] A. Shnirman and G. Schön, Phys. Rev. B **57**, 15 400 (1998).